

**IKT 495**  
**Advanced Microeconomic Theory - Fall 2011 - Final**  
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NAME: \_\_\_\_\_

*ANSWERS*

ID NO: \_\_\_\_\_

You have 110 minutes to complete the exam. NO CALCULATORS. You should write your answers in English. Numerical examples will not be counted as proofs. Turn off your cell phones and put them away. Put any class related document away. When necessary, you can write approximate answers by rounding the numbers. GOOD LUCK!

1. (15 Points) Please indicate whether each statement below is true or false OR fill in the blanks OR choose the correct one among the options written *italic*. *You do not need to explain your answers. You can write your answers on this page or on a separate page.*

- F a) If a tennis player were truly randomizing over which half of the court to serve to, she would never serve to the same side of the court 5 times in a row.
- T b) In practice, people tend to accept too many large risks (such as forgoing life insurance) and avoid too many small risks (such as losing or breaking a cell phone).
- T c) People find it easier to commit to saving for retirement from next year's salary as opposed to current salary. This behavior is consistent with hyperbolic discounting of income.
- d) Campus "Public Interest Research Groups" receive more operating funds from student registration fees when the default choice is to contribute but a box must be checked to opt out versus a default of not contributing but checking a box to contribute. This can be best explained by the concept of *Framing Effect*.
- e) People who have recently become unemployed often believe the national unemployment rate is higher than it actually is. Behavioral economists would attribute this belief to *the anchoring effect / (the law of small numbers) / time inconsistency of preferences / loss aversion / hyperbolic discounting*.
- f) In an experiment, a group of people were given the following choice: A) Lose \$100 with certainty, or B) 50% chance to win \$50, 50% chance to lose \$200. An overwhelming majority chose option B. This can be best explained by the concept of *loss aversion*...
- T g) In a student-professor matching problem, if professors do not have any preference over the students, then this becomes an assignment problem. If they do have preference, then the problem is called marriage problem.
- F h) Adverse selection occurs when an agreement encourages undesirable behavior.
- T i) There can be found a specific matching that is efficient but not stable.
- F j) Outcome of DAA (Deferred Acceptance Mechanism), gives always a unique stable matching even if the proposer roles are changed.
- F k) Nash Bargaining solution may choose a weak pareto optimal point from the feasible set.
- T l) The feasible set,  $S$ , is defined as follows:  $S = \{(x,y) | x^2 + y^2 < 1, x \in \mathbb{R}^+, y \in \mathbb{R}^+\}$ . Then,  $S$  is convex and comprehensive
- T m) If the current state is  $x$  and we cannot generate a consensus to move to another state  $y$ , then  $x$  is pareto optimal.
- F n) The "winner's curse" refers to the fact that in a sealed-bid auction with private values, the winning bidder often pays more than he would have to in order to win the object auctioned.
- F o) If a good is sold in a Dutch auction and the bidders bid rationally, the price paid for the good will always be equal to the second-highest willingness to pay of auction participants.

2. (10 points) Assume that there is a rewarding task Ali has wishes to complete today or on one of the following three days (today  $t=0$  and  $t=1,2,3$  for the following days). The task's reward is 10, 36, 50, 91 if he completes it on  $t=0,1,2,3$  respectively (This means that if he completes the task today,  $t=0$ , for example, its reward is 10; if he completes the task on  $t=2$ , its reward is 32). Exponential discount function is  $D_E(t)=0.5^t$  and hyperbolic discount function is  $D_H(t)=(1/(1+2t))$ . If Ali has  $D_E(t)$ , at which period does he finish the task? If Ali has  $D_H(t)$ , at which period does he actually finish the task and is he time consistent in completing the task? Show all your work.

	$t=0$	$t=1$	$t=2$	$t=3$	
$D_E(t)(t=0)$	10	36	50	91	$\rightarrow$ the highest one is 18. So, $t=1$ .
$D_E(t)(t=1)$	10	18	12.5	91/8	
$D_H(t)(t=0)$	10	12	10	13	$\rightarrow$ at $t=0$ , he decides to finish at $t=3$
$D_H(t)(t=1)$		36	50/3	91/5	$\rightarrow$ the highest one is 36. So, he changes his mind and complete it at $t=1$ . He is <u>time inconsistent!</u>

3. (12 points) Mr. Cem has two children, Ali and Banu. Ali is a slow learner and Banu is very bright. If Mr. Cem spends  $\$X$  per month on Ali's education, Ali will score a total of  $X/4$  points on his ÖSS tests. If Mr. Cem spends  $\$Y$  per month on Banu's education, she will score a total of  $Y$  on her ÖSS tests. Mr. Cem has a utility function  $U(A,B)$ , where  $A$  is Ali's ÖSS score and  $B$  is Banu's ÖSS score (maximum ÖSS score is 500). Mr. Cem's budget per month for their children's education is  $\$500$ . To maximize his utility, how much should Mr. Cem spend on Ali's and Banu's Education if a)  $U(A,B) = \min\{A, B\}$ ; b)  $U(A,B) = \max\{A, B\}$ ; c)  $U(A,B) = A+B$ ; d)  $U(A,B) = AB$ .

a) leximin (egalitarian) equate their scores.

$$A = 400, B = 100$$

b) and c) gives the same result as utilitarian

$$A = 0, B = 500$$

d) Nash  $A = B = 250$ .

4. (10 points) Harun and Mehmet burn with hatred for each other. They both consume apple. Harun's utility function is  $U_H = W_H - W_M^2/8$  and Mehmet's utility is  $U_M = W_M - W_H^2/8$ , where  $W_H$  is Harun's apple consumption and  $W_M$  is Mehmet's apple consumption, measured in kilograms. Muhtar currently has all the apples, a total of 58 kilograms which he could give back to them. For some reason, Muhtar wants them both to be as happy as possible and he wants to treat them equally. How should Muhtar allocate apples? (Muhtar does not have to give all the apples to them. If he does not want to give all the apples to them, he will trash the rest.)

egalitarian Muhtar.

$$U_H = U_M \Rightarrow W_H = W_M$$

$$\max_{\{W_H\}} U_H = W_H - \frac{W_M^2}{8} = W_H - \frac{W_H^2}{8}$$

$$\text{FOC} \quad 1 - \frac{W_H}{4} = 0 \Rightarrow W_H^* = W_M^* = 4.$$

So, Muhtar gives 4 apples to each, trash the rest 50 apples.

5. (10 points) A Borda rule is used to decide an election between three candidates,  $x$ ,  $y$ , and  $z$ , where a score of 2 is awarded to a first choice, 1 to a second choice, and 0 to a third choice. There are 23 voters. 3 voters rank the candidates  $x$  first,  $y$  second, and  $z$  third; 5 voters rank the candidates  $x$  first,  $z$  second, and  $y$  third; 10 rank the candidates  $z$  first,  $y$  second, and  $x$  third; and 5 voters rank the candidates  $y$  first,  $z$  second, and  $x$  third. A) Which candidate wins according to Borda rule? B) Which candidate wins according to (Condorcet) majority rule? Show all your work.

	3	5	10	5	
$x$	$x$	$z$	$y$	$y$	(2)
$y$	$z$	$y$	$z$	$z$	(1)
$z$	$y$	$x$	$x$	$x$	(0)

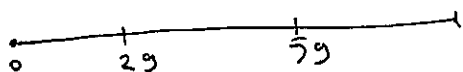
Borda :

$$\left. \begin{array}{l} x : 16 \\ y : 10 + 13 = 23 \\ z : 20 + 10 = 30 \end{array} \right\} z > y > x, z \text{ wins.}$$

Condorcet :

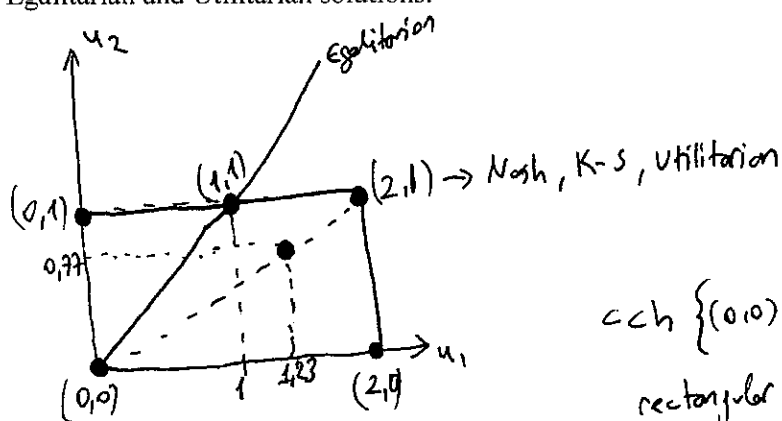
$$\left. \begin{array}{l} x < y \\ y < z \\ x < z \end{array} \right\} z > y > x, z \text{ wins.}$$

6. (12 Points) There are two bidders in a first price, sealed bid auction. Bidder 1 knows that individual 2 will submit a bid of \$29 with probability  $2/3$  and \$59 with probability  $1/3$ . Under each of the following four assumptions, calculate individual 1's payoff maximizing bid, determine the probability of person 1 winning the asset and calculate bidder 1's payoff. A) Bidder 1's reservation value is \$99; B) Bidder 1's reservation value is \$150; C) Bidder 1's reservation value is \$40; D) Bidder 1's reservation value is \$22; ~~the value of the asset is \$100~~. (Bidder 1 is risk neutral and bidding increments is \$1.)



- a) Bidder 1's res. value: 99  
 He either  ~~bids~~ bids 60 or 30. If he bids 60, he wins for sure his return is  $99 - 60 = 39$ . If he bids 30, he wins with prob.  $2/3$  loses with prob.  $1/3$ , exp. return =  $2/3 \cdot (99 - 30) = 46$ . So he bids 30.
- b)  $v_1 = 150$   
 bid 60  $\rightarrow$  win for sure, payoff =  $150 - 60 = 90$   
 bid 30  $\rightarrow$  win  $p = 2/3$ , " =  $2/3 \cdot (150 - 30) = 80$  } optimal bid = 60!
- c)  $v_1 = 40$ , he does not bid more than 40. So bid 30.  
 win  $p = 2/3$ , payoff =  $2/3 \cdot (40 - 30) = 20/3$
- d)  $v_1 = 22$ , bid  $b \leq 22$ , lose anyway.

7. (12 Points) Let the utility possibility set  $S$  in  $\mathbb{R}^2$  be the convex and comprehensive hull of the points  $(0,0)$ ,  $(0,1)$ ,  $(2,1)$  and  $(0.77, 1.23)$ . For this  $S$ , compute Kalai-Smorodinsky, Nash, Egalitarian and Utilitarian solutions.



$$cch \{(0,0), (0,1), (2,1), (0.77, 1.23)\} = \text{rectangle} \rightarrow \{(0,0), (0,1), (2,1), (2,0)\}$$

So, egalitarian chooses  $(1,1)$   
 Nash, K-S and Utilitarian choose  $(2,1)$ .

8. (14 points) Consider the set of men  $\{A, B, C, D\}$  and women  $\{X, Y, Z\}$  with the below preferences.

A	B	C	D	X	Y	Z
X	Y	X	Z	A	D	B
Y	X	Z	X	C	A	A
	Z		Y	B		D
				D		

- a) (2 points) Write a matching which is not stable. Explain briefly.  
 b) (4 points) Find men optimal stable matching. Show every step in the relevant algorithm.  
 c) (4 points) Find women optimal stable matching. Show every step in the relevant algorithm.  
 d) (4 points) Write an unstable but efficient matching. Explain why?

a)  $\{(A, Y), (B, X), (D, Z), C\}$

b)  $\{(A, X), (B, Y), (C, X), (D, Z)\}$   
 $\begin{matrix} + & - & - & + \\ (B, X) & (C, Z) & (D, X) & \\ - & - & - & \\ (B, Z) & C & (D, Y) & \\ + & + & + & \end{matrix}$   $\left\{ (A, X); (B, Z); C; (D, Y) \right\}$

men optimal

the same

c)  $\{(X, A); (Y, D); (Z, B); C\}$  women optimal

d)  $\Pi = \{(A, Y), (B, Z), (C, X), D\}$  not stable due to  $(A, X)$

- the only way we break  $(A, Y)$  is to move to  $(A, X)$  but  $C$  gets worse off (and  $Y, D$ )
- If we break  $(B, Z)$ ,  $Z$  gets worse off.
- " " "  $(C, X)$ ,  $C$  " " " " " " " " " "

∴  $\Pi$  is unstable but efficient!