İKT 495 - Advanced Microeconomic Theory - Fall 2010 - Final

Zafer Akın

December 23, 2010

ANSWERS

NAME: GOOD LUCK!

You have 110 minutes to complete the exam. **NO CALCULATORS.** You should write your answers in-English. **Numerical examples will not be counted as proofs. Turn off your cell phones** and put them away. **Put any class related document away.** When necessary, you can write approximate answers by rounding the numbers.

1. (18 Points) Please indicate whether each statement below is *true* or *false* OR fill in the blanks. You do not need to explain your answers. You can write your answers on this page or on a separate page.

T *a.* If you, as the human resource manager of a firm, should decide on how many mutual funds to offer in your employees' pension plan, it may be a good idea to offer only 10 choices rather than 50 choices.

T *b.* In a student-professor matching problem, if professors do not have any preference over the students, then this becomes an assignment problem. If they do have preference, then the problem is called marriage problem.

T *c.* If a college degree is used to signal high ability, but the cost of a college degree is very high, a pooling equilibrium is achieved.

F *d.* "US still continues the war in Iraq although they accept that they should not". This is an example of a phenomenon called "Loss Aversion".

e. In a lottery experiment, group 1 is told that they have a 55% chance of winning, group 2 is told they have a 45% chance of losing. Group 1 is more likely to buy this lottery.

F *f.* Adverse selection occurs when an agreement encourages undesirable behavior.

T *g.* English auctions always give pareto efficient outcomes.

T *h.* There can be found a specific matching that is efficient but not stable.

T *i.* In ultimatum game, it is optimal to offer a minimal amount to responder. However, in experiments, proposers offer much more than the minimal share.

F *j.* Outcome of DAA (Deferred Acceptance Mechanism), gives always a unique stable matching even if the proposer roles are changed.

F *k.* Nash Bargaining solution may choose a weak pareto optimal point from the feasible set.

l. If people's choices are affected by how they are presented, this is called framing effect

T *m.* The feasible set, S, is defined as follows: $S = \{(x,y)|x^2 + y^2 < 1, x \in \mathbb{R}^+, y \in \mathbb{R}^+\}$. Then, S is convex and comprehensive

F *n.* Men are more overconfident than women such that men trade much less than women in stock exchanges.

F *o.* In second price (Vickrey) auctions, if you are risk averse, it is optimal to bid less than your own valuation.

T *p.* Kalai-Smorodinsky Bargaining solution does not satisfy contraction independence.

F *r.* If sellers of good cars and sellers of lemons both offer a warranty on their car, consumers will then be able to tell which cars are the lemons.

s. If you keep postponing doing your homework, this is an example of ....time inconsistency

2. (14 points) *a)* What is sunk cost fallacy? Give a simple example. *b)* Assume that there is a costly task Ali has to complete today or on one of the following three days (today $t = 0$ and $t = 1, 2, 3$ for the following days). The task's cost is 10, 18, 32, 54 if he completes it on $t = 0, 1, 2, 3$ respectively (This means that if he completes the task today, $t = 0$, for example, its cost is 10; if he completes the task on $t = 2$, its cost is 32). Exponential discount function is $D^E(t) = 0.5^t$ and hyperbolic discount function is $D^H(t) = \frac{1}{1+2t}$. If Ali has $D^E(t)$, at which period does he finish the task? If Ali has $D^H(t)$, at which period does he actually finish the task and is he time consistent in completing the task? Show all your work.

3. (16 points) Consider a lovely family; Ali (father), Bahar (mother) and their kid Cem. They love Cem very much but they experience a problem about getting Cem to go to sleep at night. If his father is together with Cem, he sleeps in t_f minutes; with his mother, he sleeps in t_m minutes. Since this is a costly task, Ali and Bahar both get disutility from it. The utility functions are given as $u_{Ali}(t) = -t^a$ and $u_{Bahar}(t) = -t^b$ (t represents minutes and $1 \leq a, b$). They are supposed to agree upon who will get the kid to go to sleep for T days (for example, they each take $\frac{T}{2}$ days).
a) (4 points) Given this information, write down the problems that you should solve if you want to suggest Ali and Bahar the allocation of T day task between them based on Egalitarianism, Utilitarianism and Nash Bargaining? (Suppose $T_{Ali} + T_{Bahar} = T$, or $T_{Bahar} = T - T_{Ali}$. Suppose also that there is no discounting and total disutility of Ali from T_{Ali} days of task is given by $T_{Ali}.u_{Ali}(t) = -T_{Ali}.t^a$, similar for Bahar.)
b) (4 points) Solve the utilitarian problem when $t_f = 15$, $t_m = 55$, $a = 3$, and $b = 2$. Solve the problem again when $t_f = 20$, $t_m = 20$, $a = 3$, and $b = 2$. Give an intuitive explanation for your answer.
c) (5 points) Solve the egalitarian problem when $t_f = 20$, $t_m = 20$, $a = 3$, and $b = 2$.
d) (3 points) What is the Nash solution? Give an intuitive explanation for your answer.

4. (16 Points) Consider a two-person bargaining solution, $\mathcal{F}$, on convex and comprehensive sets, denoted by S, in R^2_+. Define the ideal point $a_i(S)$ of S for agent $i = 1, 2$ as $a_i(S) = \max\{x_i | x \in S\}$. $F(S)$ is the middle point of the line segment connecting the ideal points, defined by $\mathcal{F}_i(S) = a_i(S)/2$. Show whether the solution $\mathcal{F}$ satisfies:
a) (4 pts) (weak/strong) Pareto optimality
b) (4 pts) scale invariance
c) (4 pts) strong monotonicity
d) (4 pts) contraction independence.

5. (20 points) Consider the set of men $\{m_i\}_{i=1}^3$ and women $\{w_i\}_{i=1}^3$ with the below preferences.

$$\begin{aligned} R^{m_1} &= w_2, w_1, w_3 \\ R^{m_2} &= w_1, w_3, w_2 \\ R^{m_3} &= w_1, w_3, w_2 \\ R^{w_1} &= m_2, m_3, m_1 \\ R^{w_2} &= m_2, m_1, m_3 \\ R^{w_3} &= m_1, m_2, m_3 \end{aligned}$$

a) (5 points) Write a matching which is not stable different than the matching given in part "c". Explain briefly.
b) (10 points) Find a stable matching. Show every step in the relevant algorithm.
c) (5 points) Prove that the unstable matching $\pi = \{(m_1, w_2), (m_2, w_3), (m_3, w_1)\}$ is efficient.

Choose one of the following questions, 6.A or 6.B (only solve one of them; if you do both, indicate which one you want to be graded on, otherwise one of them will be chosen randomly):

6.A (16 Points) Consider a sealed bid first price auction with just two bidders. Suppose that the bidders' valuations are independent and uniformly distributed on [0, 10]. Suppose that bidder 2 is a bit stupid and always bids his true valuation, and that bidder 1 knows this. Given bidder 2's behaviour, what is bidder 1's optimal bid, b_1^*, as a function of her true value, V_1? Does knowing the information that bidder 2 always bids his true valuation have an effect on bidder 1's bidding strategy? Why? Comment on your answer.

6.B (16 Points) A single, indivisible object is sold by auction mechanism. There are three bidders and the auctioneer (owner of the object) thinks that there may be *three cases of bidder valuations, with equal probability* 1/3, that are as follows:

Prob	1/3	1/3	1/3
	Case A	*Case B*	*Case C*
Bidder 1	80	75	90
Bidder 2	100	100	130
Bidder 3	60	140	160

Based on this table, which auction is more profitable for the auctioneer, vickrey or first price auction? Show all your calculations. What would be your answer if '160' in the table is replaced by '380'.

IKT 695 - Fall 2010 - FINAL ANSWERS

2) a) see lecture notes

b)

t=	0	1	2	3
cost	10	18	32	54

} costly task (choose minimum cost)

$D^E(t) = 0.5^t$, $D^H(t) = \frac{1}{1+2t}$

Exponential: present discounted values

$10, 9, 8, \frac{54}{8}$ so Exp. agent does the task at $t=3$

Hyperbolic at $t=0$

present discounted values

$10, \frac{18}{3}, \frac{32}{5}, \frac{54}{7}$ at $t=0$, she decides to complete the task at $t=1$.

at $t=1$

present discounted values (P.D.V.)

$18, \frac{32}{3}, \frac{54}{5}$ at $t=1$, she decides to complete the task at $t=2$

at $t=2$

P.D.V.

$32, \frac{54}{3}$ at $t=2$, she postpones it again till $t=3$

This, she completes the task at $t=3$! she is time-inconsistent!

3) a) Utilitarian: $\max_{\{T_{ALi}\}} \left(-T_{ALi} \cdot t_f^a - (T - T_{ALi}) \cdot t_m^b \right)$

Egalitarian: $-T_{ALi} \cdot t_f^a = -(T - T_{ALi}) \cdot t_m^b$

NASH: $\max_{\{T_{ALi}\}} \left(-T_{ALi} \cdot t_f^a \right) \cdot \left(-(T - T_{ALi}) \cdot t_m^b \right)$

b) if $t_f^a > (<)\ t_m^b$, $T_{ALi} = 0$ $(T_{Bohr} = 0)$

since $t_f = 15$, $t_m = 55$, $a = 3$ and $b = 2$; and $15^3 > 55^2$, $T_{ALi} = 0$, $T_{Bohr} = T$

If $t_f = t_m = 20$ and $a = 3$ and $b = 2$; The result is the same.

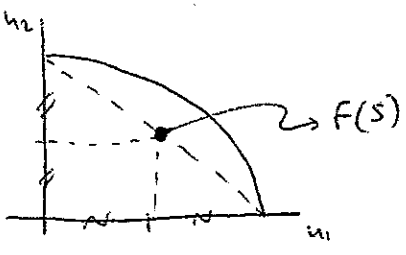
Utilitarian chooses the one whose utility is less to maximize total utility!

3-c) Egalitarian $T^*_{Ali} = \frac{T \cdot t_m^b}{t_f^a + t_m^b}$

$T^*_{Ali} = \frac{T \cdot 20^2}{20^3 + 20^2} = \frac{T}{21}$, $T^*_{Schw} = \frac{20T}{21}$.

d) NASH $T^*_{Ali} = T_{Schw} = \frac{T}{2}$!

4)



a) NOT P.O. !
b) Scale Invariant !
c) Strongly monotone
d) NOT Contraction independent!

5) a) $\{(m_1, w_3), (m_2, w_2), (m_3, w_1)\}$ not stable since m_2 and w_1 breaks this matching.

b) $\{(m_1, w_2), (m_2, w_1), (m_3, w_3)\}$ is stable.

c) $\{(m_1, w_2), (m_2, w_3), (m_3, w_1)\}$

↓ if we break this m_1 gets worse off

↓ if we break this m_3 gets worse off

} So it is efficient.

6.A.) - expected profit $\Pi_e = \underbrace{(v_1 - b_1)}_{\text{profit if win}} \cdot \underbrace{\frac{b_1}{10}}_{\text{prob of winning}}$ $\max_{\{b_1\}} \Pi_e \Rightarrow b_1^* = \frac{v_1}{2}$

- it does not affect the decision because this gives no superior information about the distribution of v_2.

6.B.) Vickrey

	A	B	C
winner	2	3	3
price	80	100	130

exp. profit $= \frac{1}{3}(80 + 100 + 130) = 103.3$

1st price

	A	B	C
winner	2	3	3
P	2/3 · 100	2/3 · 140	2/3 · 160

$\Pi_e : \frac{1}{3} \cdot \frac{2}{3}(100 + 140 + 160) = \frac{800}{9}$

$\Pi_{vickrey} = 103.3 > \frac{800}{9} = \Pi_{1st\ price}$

- if 160 is 380 now, then $\Pi_{1st\ price} = \frac{1260}{9} > \Pi_{vickrey} = 103.3$.