# İKT 495 - Advanced Microeconomic Theory - Fall 2010 - Midterm

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NAME: GOOD LUCK!

You have 100 minutes to complete the exam. **NO CALCULATORS.** You should write your answers in English. **Numerical examples will not be counted as proofs. Turn off your cell phones** and put them away. **Put any lass related document away.** When necessary, you can write approximate answers by rounding the numbers.

1. (20 Points) Please indicate whether each statement below is *true* or *false* OR fill in the blanks. You do not need to explain your answer. You can write your answers on this page or on a separate page.
   *a.* ORDINAL (Ordinal or cardinal) approach to redistributive justice basically offers voting mechanisms to aggregate individual preferences.
   *b.* Discounted ticket prices for students stems from the Compensation principal.
   *c.* Reward principal suggests that past sacrifices (wrongdoings) justify larger (lesser) shares.
   true *d.* If there is free disposal of utility, then we have comprehensiveness.
   true *e.* Pareto optimality most of the time does not suggest anything about distributive justice and boils down to a no waste condition.
   False *f.* The most egalitarian sharing rule is Proportional sharing then equal surplus and the least egaliterian is the uniform gain rule.
   F *g.* Utilitarian and Leximin (egalitarian) approach pays no attention to the intensity of preferences.
   T *h.* A binary relation $\succsim$ is said to be rational if it satisfies completeness and transitivity.
   F *i.* Plurality voting rule takes into account the entire preference relation of voters over candidates.
   T *j.* Plurality rule is a special scoring rule.
   T *k.* The key for the signal to be credible in an adverse selection problem is the signal being more costly enough for one type than the other.
   F *l.* If the deductible amount is higher, you expect to pay a higher premium for the insurance.
   *m.* Moral hazard refers to a situation where there is hidden action, adverse selection refers to a situation where there is hidden information
   F *n.* In auctions, valuation ($v$) refers to buyers' (sellers') maximum willingness to pay (sell) amounts. However, $b$ refers to bid of each buyer. Moreover, they have to be the same in all auction mechanisms.
   T *o.* In second price (Vickrey) auctions, It is optimal to bid one's own valuation regardless of risk preferences.
   T *p.* Incentive compatibility simply refers to a "not mimicing others" condition (Bid your own value).
   *r.* In auctions, one of the most important aims is the outcome being efficient meaning that the one who has the highest valuation should get the asset.
   F *s.* Winners curse is more dangerous when the auction is Vickrey auction.
   T *t.* As the number of bidders increase, winners curse becomes worse.
   F *u.* If a set is compact and convex, then it has to be comprehensive.

2. (12 Points) Assume that there are five agents and their taxable incomes are 10, 15, 30, 40, 55, respectively. Their total aftertax income is $t = 100$. *a)* First compute the aftertax distribution of incomes under the basic methods, PRO (Proportional rule), UG (Uniform Gain), and UL (Uniform Loss). *b)* Now, form a table by writing down the preference of each agent over these three alternative methods. *If the agents could vote over these alternatives, which method of sharing (PRO, UG or UL) would be the winner based on three voting rule (Plurality, Condorcet, Borda)?* Write down all the preference orderings based on each voting rule (For example: first agent prefers UG over PRO and she prefers PRO over UL. After making these orderings for each agent, apply each voting rule).

3. (16 points) A husband (Ali) and wife (Bahar) would like to spend their holiday in Kurban Bayrami with their family/parents. There are 10 days of holiday (240 hours). Let $t_i$ be the hours spend in $i's$ family (We assume Ali and Ayse spend time together). Let $\delta_i(j)$ be a scaling parameter for $j's$ utility from spending time in $i's$ family where $i, j \in \{A, B\}$ where $A$ refers to Ali and $B$ refers to Bahar (for example, $\delta_A(B)$ is the scaling parameter of Bahar when she spends time with Ali' family). Then, total utility for $j$ can be formalized as $u(j) = \sum_{i \in \{A,B\}} \delta_i(j).\ln(t_i)$ (for example, $\delta_A(B).\ln(t_A)$ represents utility of Bahar from spending $t_A$ hours in Ali' family). We assume that $\delta_i(i) > \delta_i(j)$ and $\delta_i(i) > \delta_j(i)$ meaning that each gets more utility than the other from staying in his/her own family and each prefers to spend more time with his/her own family, respectively.
a) (6 points) Given this information, **Write down the problems** that you should solve if you want to suggest Ali and Bahar a time allocation between their families if you favor Egalitarianism, Utilitarianism and Nash Bargaining? (just write down the problems, you are not supposed to solve them)
b) (5 points) Solve the utilitarian problem when $\delta_B(B) = 4$, $\delta_A(A) = 3$, $\delta_B(A) = 2$ and $\delta_A(B) = 1$. Solve the problem again when $\delta_B(B) = 4$, $\delta_A(A) = 7$, $\delta_B(A) = 2$ and $\delta_A(B) = 1$. Give an intuitive explanation to your answer (note that utilitarian approach takes the scaling parameters into account).
c) (5 points) Solve the egalitarian problem when $\delta_B(B) = 4$, $\delta_A(A) = 3$, $\delta_B(A) = 2$ and $\delta_A(B) = 1$.
d) (*BONUS 1*, 3 points) Show that Nash solution gives the same result with the utilitarian solution when $\delta_B(B) = 4$, $\delta_A(A) = 3$, $\delta_B(A) = 2$ and $\delta_A(B) = 1$.

4. (10 points) What are the four elementary principals of redistributive justice? Explain each with one sentence (One of the principals has two kinds, mention them too).

5. (15 points) a) (9 points) We must divide 100 TL between two agents. Compute utilitarian, egalitarian and Nash solutions if the utility for money is ($x$ is the amount of money)
$u_1(x) = x^\beta$ and $u_2(x) = x^{1-\beta}$ $(1 > \beta > 0)$ and $\beta = 1/3$.
b) (4 points) What happens in question a when the cash prize increases (for example they share 1000 TL)? Which agent gets a bigger share of the increment according to what solutions?
c) (2 points) What happens if $\beta$ increases and What does this imply in terms of allocations?

6. (15 points) a) (3 points) Take the following profile with 55 voters and five outcomes $(a, b, c, d, e)$. Which candidate is the winner according to the plurality voting rule?

| Number of voters: | 18 | 12 | 10 | 9 | 4 | 2 | |
|---|---|---|---|---|---|---|---|
| | $a$ | $b$ | $c$ | $d$ | $e$ | $e$ | 4 |
| | $e$ | $e$ | $b$ | $c$ | $b$ | $c$ | 3 |
| | $d$ | $d$ | $e$ | $e$ | $d$ | $d$ | 2 |
| | $c$ | $c$ | $d$ | $b$ | $c$ | $b$ | 1 |
| | $b$ | $a$ | $a$ | $a$ | $a$ | $a$ | 0 |

b) (4 points) Compute the majority relation and show that there is no cycle. c) (5 points) Compute the Borda ranking and show that it orders the candidates exactly like the majority relation.d) (3 points) State Arrow's impossibility theorem by mentioning its axioms.

7. (12 points) a) (5 points) There are two types of workers, H and L. The marginal product of an H type is 2m and the value of the marginal product of an L type is m. An employer cannot directly distinguish an H type from an L type. What type of a problem exists in this market (Adverse selection or moral hazard)?
b) (5 points) Suppose education can be used as a signal of type in this market. Suppose that the cost of $e$ units of education to an H type is $c_h = \frac{e^2}{2}$ and the cost of $e$ units of education to an L type is $c_l = e^2$. Find a separating equilibrium and characterize it (Find a level of education such that the H type chooses to acquire education and the L type does not). Assume $m = 9$. What is the range of $e^*$ (the level of education that provides separation of types in equilibrium). Which level of education type H should choose?
c) (2 points) What happens if $c_l = c_h = e^2$. Can there be a separating equilibrium?

*BONUS 2* (4 points): Show that in a first price auction, a bidder bids more if he/she is risk averse than if he/she is sk neutral (assume private values, valuations are $v_1$ and $v_2$, $n = 2$ and $v_2 \sim [0, 1]$ uniform distribution).

$\sum x_i = 150$

2) $x_i = 10, 15, 30, 40, 55$ } deficit case ($T - \sum x_i = 50$)

$T = 100$.

| | 10 | 15 | 30 | 40 | 55 | |
|---|---|---|---|---|---|---|
| PRO : | 6.66 | 10 | 20 | 26.6 | 36.6 | (multiply $x_i$ with $\frac{100}{150}$) |
| UL : | 0 | 5 | 20 | 30 | 45 | (subtract 10 from each one) |
| UG: | 10 | 15 | 25 | 25 | 25 | ($\frac{100}{5} = \lambda = 20 \rightarrow$ 1 and 2 gets their orjinal shares. $100-10-15 = 75/3 = \lambda' = 25$. others get 25. |

| 1. | 2. | 3. | 4. | 5. agent |
|---|---|---|---|---|
| UG | UG | UG | UL | UL |
| PRO | PRO | UL | PRO | PRO |
| UL | UL | PRO | UG | UG |

Plurality : U.G.

Borda : UG > UL > PRO

Condorcet : UG > UL, UG > PRO, PRO < UL } UG > UL > PRO

3) a) utilitarian → $\max_{\{t_A, t_B\}}$ $u(Ali) + u(Bahar)$ subject to

$u(Ali) = \delta_A(A).\ln(t_A) + \delta_B(A)\ln(t_B)$ $\quad t_A + t_B = 240$

$u(Bahar) = \delta_B(B)\ln(t_B) + \delta_A(B)\ln(t_A)$

Egalitarian $\Rightarrow$ $u(Ali) = u(Bahar)$

Nash $\Rightarrow$ $\max_{\{t_A, t_B\}}$ $u(Ali) \times u(Bahar)$ s.t. $t_A + t_B = 240$

b) utilitarian $\Rightarrow$ $t_A^* = \dfrac{240\,(\delta_A(A) + \delta_A(B))}{\delta_A(A) + \delta_A(B) + \delta_B(A) + \delta_B(B)} = 96$ hours ; $t_B^* = 144$ hours

$t_A^* = \dfrac{240 \cdot 8}{14} \simeq 137$ hours ; $t_B^* = 93$ hours

c) Egalitarian $\Rightarrow$ $[\delta_A(A) - \delta_A(B)]\ln(t_A) = [\delta_B(B) - \delta_B(A)]\ln(240 - t_A)$

$2\ln(t_A) = 2\ln(240 - t_A) \Rightarrow t_A^* = t_B^* = 120$

d) NASH $\Rightarrow$ $\max_{\{t_A, t_B\}} \ln(t_A^3 t_B^2) \cdot \ln(t_B^4 \cdot t_A) \Rightarrow t_A^* = 96$ ; $t_B^* = 144$

4) lecture notes

5) a) utilitarian $\Rightarrow \max_{\{x\}} x^{1/3} + (100-x)^{2/3} \rightarrow$ FOC gives $x^* \cong 3.5$, $100-x^* \cong 96.5$

Egalitarian $\Rightarrow x^{1/3} = (100-x)^{2/3} \Rightarrow x^* \cong 90.5$; $100-x^* = 9.5$

NASH $\rightarrow \max x^{1/3}(100-x)^{2/3} \Rightarrow$ FOC: $x^* = \frac{100}{3}$; $100-x^* = \frac{200}{3}$

b) utilitarian $\Rightarrow x^*$ increases proportional to prize's square root

Egalitarian $\Rightarrow x^*$ " much more than the other.

NASH $\Rightarrow$ proportional sharing

c) let $p: \frac{1}{3} \rightarrow {}^2/_3$. the shares above will be replaced.

6) a) plurality: a

b) Condorcet: $a<b$ $a<c$ $a<d$ $a<e$
$b<c$ $b<d$ $b<e$
$c<d$ $c<e$
$d<e$

$e>d>c>b>a$

no cycle!

c) a: 72
b: 101
c: 107
d: 119
e: 152

$e>d>c>b>a$

d) see lecture notes.

7) a) Adverse selection.

b) benefit: $2m-m$ cost: $e^2/2$ benefit $= 2m-m$ cost $= e^2$ $\Rightarrow m > \frac{e^2}{2}$ and $m < e^2$

$\Rightarrow e^2 > m > \frac{e^2}{2}$ or $\sqrt{2m} > e^* > \sqrt{m}$

If $m=9 \Rightarrow 3\sqrt{2} > e^* > 3$. H should choose $e=3$!

c) if $c_L = c_H = e^2$, then there can't be separating equilibrium.

Bonus 2: assume $u(x) = x^\alpha$, $0<\alpha<1$. then

$\max_{\{b_1\}}$ expected profit $= (v_1 - b_1)^\alpha \cdot b_1 + 0 \cdot (1-b_1) \Rightarrow$ F.O.C $b_1^* = \frac{v_1}{1+\alpha} > \frac{v_1}{2}$ (bid for a risk neutral agent)

Q.E.D.